

Andra Oltmann\*, Eike Petersen, Roman Kusche, and Philipp Rostalski

# Investigation of an Acceleration Pipeline for Single Fiber Action Potential Simulation

<https://doi.org/10.1515/cdbme-2022-1069>

**Abstract:** Surface electromyography (sEMG) records the electrical potentials on the skin surface generated by the electrical activity of muscle fibers. To better understand physiology and to assess signal processing algorithms, sEMG models have been developed in the past. However, numerical models required for modeling accurate geometries very often have a high computational complexity rendering realistic simulations challenging. In order to cope with this challenge, a pipeline is investigated to accelerate the calculation of the single fiber action potential (SFAP) based on the principle of reciprocity. To enable a comparison to an analytical solution, the investigation is carried out in a highly simplified muscle model. The results show that the pipeline is suitable for calculating SFAP with a low mean absolute error sufficient for application in sEMG models. In addition, there is a significant reduction of the calculation time allowing the simulation of even highly complex geometries and a large number of muscle fibers.

**Keywords:** numerical computer simulation, muscle modelling, sEMG, single fiber action potential

## 1 Introduction

Surface electromyography (sEMG) records the electrical potentials on the skin surface generated by the electrical activity of muscle fibers during contraction [1]. The areas of application in which sEMG is used are manifold. These include motion analysis using sEMG signals from the extremities [1], but also the estimation of inspiratory patient effort under mechanical ventilation based on sEMG signals derived from the torso [2].

To better understand the underlying physiology and to assess novel signal processing algorithms, sEMG models have been developed in the past, e.g. [3–5]. The basis of these models is the calculation of the single fiber action poten-

tials (SFAPs), which are subsequently superimposed to simulate the sEMG interference signal. Anatomical and numerical models can be distinguished [1]. Analytical models are characterized by their fast calculation, but can only be solved for simplified geometries [1, 4]. In contrast to this are the numerical models, in which accurate anatomies and inhomogeneities can be integrated. However, the disadvantage is the high computational effort when approximating the SFAP [1, 3]. This becomes particularly significant when simulating SFAPs for muscles with a large number of fibers.

To solve this problem of complexity, *Pereira Botelho et al.* [3] proposed to apply the principle of reciprocity in which the behavior of the volume conductor is simulated for each electrode instead of each fiber. Based on this, the objective of this work is to investigate a pipeline to accelerate the SFAP computation. The proposed pipeline in this work is applied to a simplified muscle model in order to verify the results by comparison with an analytical SFAP solution.

## 2 Methods

### 2.1 Analytical SFAP model

In order to have a ground truth for the calculation of SFAP, the analytical volume conductor model proposed by *Farina et al.* [6] and applied in *Petersen et al.* [4] is used, code available at [7]. This model has a three layer geometry. At the bottom is a planar, infinitely extended muscle layer. This is covered by a planar, infinitely extended fat and skin layer. The muscle fiber runs in parallel to the skin surface and the electrodes are arranged on this top layer.

In order to determine the electric potential in the volume conductor, it is necessary to solve the forward problem. It can be assumed that the electric field behaves like a static field at all points in times because the physiological system has a low rate of change [8]. With this quasi-static condition, a solution for the electric potential  $\phi$  can be calculated using Poisson's equation, where  $\sigma$  denotes the electrical conductivity and  $I_{vc}$  a current density source [8]:

$$-\sigma \cdot \nabla^2 \phi = I_{vc}. \quad (1)$$

A purely resistive behavior is assumed. The cause of the electrical source  $I_{vc}$  is the action potential. This is generated at

\*Corresponding author: **Andra Oltmann**, Fraunhofer IMTE, Mönkhofer Weg 239a, 23562 Lübeck, Germany, e-mail: [andra.oltmann@imte.fraunhofer.de](mailto:andra.oltmann@imte.fraunhofer.de)

**Eike Petersen**, Technical University of Denmark, DTU Compute, Kgs. Lyngby, Denmark

**Roman Kusche**, Fraunhofer IMTE, Lübeck, Germany

**Philipp Rostalski**, Fraunhofer IMTE, Lübeck, Germany and Institute for Electrical Engineering in Medicine, Universität zu Lübeck, Lübeck, Germany

the neuromuscular junction (NMJ), propagates from the NMJ towards the fiber ends and is finally extinguished at these ends [1]. The source signal  $I_{vc}$  used for the methods represents the transmembrane current and is the one proposed by Farina et al. [6], in the formulation proposed by Petersen et al. [4], with an additional scaling factor according to the core conductor model [1]:

$$I_{vc}(x, t) = \sigma_i \cdot \pi \cdot r^2 \cdot \left[ EOF_1(t) \delta(x - x_n - L_1) + \psi'(x - x_n - vt) p_1(x) + GEN(t) \delta(x - x_n) + \psi'(-x + x_n - vt) p_2(x) + EOF_2(t) \delta(x - x_n + L_2) \right] \quad (2)$$

In this equation,  $\psi = \frac{\partial}{\partial x} V_m(-x)$  describes the voltage gradient across the fiber membrane, where  $V_m$  is the *Rosenfalck* [9] analytical description of the action potential. The Dirac distribution is described by  $\delta$ , the intracellular conductivity by  $\sigma_i$ , the fiber radius by  $r$ , the propagation velocity by  $v$ , the position of the NMJ by  $x_n$  and the length of the fiber ends by  $L_1$  and  $L_2$ .  $p_1(x)$  and  $p_2(x)$  are the characteristic functions of the fiber halves [4]. In the source term  $I_{vc}$  the generation component of the action potential is described by

$$GEN(t) = 2 \cdot \psi(-vt) \quad (3)$$

and the end-of-fiber components by

$$EOF_1(t) = -\psi(L_1 - vt) \quad (4)$$

and

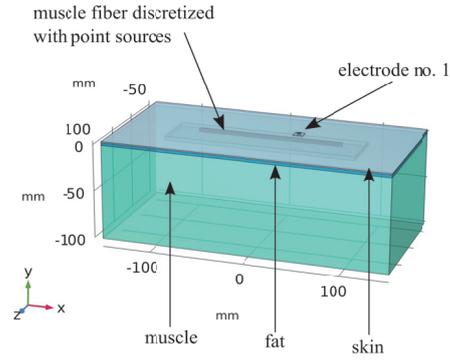
$$EOF_2(t) = -\psi(L_2 - vt). \quad (5)$$

The SFAP is calculated by convolving the two-dimensional analytical global transfer function of the volume conductor with the described current density source [6], with a numerical integration used in [4].

## 2.2 Numerical SFAP model

The investigation of the proposed numerical computation acceleration pipeline starts with the transfer of the analytical model into a numerical model (method no. 1). The geometry used for the simulation is shown in Figure 1 and the corresponding properties are listed in Table 1. The main difference to the volume conductor of the analytical model is that the tissue layers are not infinitely extended. Nevertheless, experiments have shown that they are chosen large enough to be of negligible influence.

In the numerical model, the fiber is discretized using 2000 point sources between NMJ and fiber end, as proposed in [5]. Depending on the spatial position of the point source along the fiber, the corresponding signal component of  $I_{vc}$  (see



**Fig. 1:** Simplified muscle model with muscle (green), fat (dark blue) and skin (light blue) layer as well as used electrode position. The muscle fiber runs in parallel to the skin surface and is discretized with point sources

**Tab. 1:** Properties of the implemented geometry (C: center, L: length, H: height, W: width)

Object	Properties
Muscle	C: $x = 0$ mm, $y = -50$ mm, $z = 25$ mm, $L = 30$ cm, $H = 100$ mm, $W = 15$ cm
Fat	C: $x = 0$ mm, $y = 1.5$ mm, $z = 25$ mm $L = 30$ cm, $H = 3$ mm, $W = 15$ cm
Skin	C: $x = 0$ mm, $y = 3.5$ mm, $z = 25$ mm $L = 30$ cm, $H = 1$ mm, $W = 15$ cm
NMJ	$x = -0.95$ mm, $y = -1.58$ mm, $z = 12.04$ mm
L1	75.86 mm
L2	73.97 mm
Electrode no. 1	$x = 30$ mm, $y = 4$ mm, $z = 10$ mm

equation 2) is applied. The propagating signal component must be integrated over the discretized fiber section. For simplification, the current value at the position of the point source is multiplied with the length of the fiber section.

In order to solve the governing equations using the finite element method (FEM), initial and boundary conditions are required. The initial condition  $\phi_0 = 0$  V applies to all regions of the numerical model. At the bottom of the muscle layer the grounding is established. Electrical insulation is applied to all other outer layers of the tissues. COMSOL Multiphysics® v. 5.6 (COMSOL AB, Stockholm, Sweden) is used for modelling. The simulation parameters for the time-dependent study are given in more detail in section 2.4.

## 2.3 Acceleration pipeline for numerical models

To reduce the computational cost of the SFAP calculation, *Pereira Botelho et al.* [3] proposed a method which will be

studied in detail with the simplified muscle geometry. In particular, it is assumed that the principle of reciprocity can be applied to the linear system. This indicates that the electrical potentials  $\phi$  within the volume conductor are calculated for each electrode instead of each fiber [3].

### 2.3.1 Reciprocal SFAP calculation by means of transfer function

The next method of the acceleration pipeline proposed in this work (method no. 2) uses the principle of reciprocity and calculates the time-depending transfer function from the electrode to a point along the fiber. For this a current curve in form of a ramp function  $r(t)$  is applied to the electrode on the skin surface:

$$r(t) = \begin{cases} 0 \text{ [A]}, & \text{if } t \leq 0 \text{ s} \\ 100 \left[\frac{\text{A}}{\text{s}}\right] \cdot t, & \text{if } 0 \text{ s} < t < 0.01 \text{ s} \\ 1 \text{ [A]}, & \text{if } t \geq 0.01 \text{ s}. \end{cases} \quad (6)$$

The electrical potentials are calculated time-dependently for the entire volume conductor by means of the Poisson equation using FEM in COMSOL Multiphysics<sup>®</sup>. All previously defined initial conditions and boundary conditions are maintained. In a post-processing step, COMSOL Multiphysics<sup>®</sup> LiveLink for MATLAB first determines the coordinates of the muscle fiber in the volume conductor (2000 coordinates between NMJ and fiber end). For each of these coordinates, a potential curve  $\phi(t)$  was calculated beforehand using FEM. With the given current curve  $r(t)$ , the transfer function can thus be determined for each coordinate. Since this can also be applied reciprocally, each is convolved with the corresponding  $I_{vc}$  signal component (see equation 2) depending on the spatial position along the fiber. The final SFAP is obtained by superposition of all convolution results of the individual fiber points.

### 2.3.2 Reciprocal SFAP calculation by means of weighting function

The third method (method no. 3) is the one originally proposed by *Pereira Botelho* et al. [3]. It is additionally assumed that due to the purely resistive properties the temporal low-pass behavior is negligible. Due to the reciprocity principle, a unit current source is applied to the electrode [3]. Using the known Poisson's equation, the initial and boundary conditions, the electrical potentials for a steady-state study are calculated using COMSOL Multiphysics<sup>®</sup>. This results in a gain factor for each point along the fiber as the transfer response. In post-processing with the COMSOL Multiphysics<sup>®</sup> LiveLink for

**Tab. 2:** Applied model parameters

Parameter	Value
<b>Electrical conductivities at 150 Hz used in [3]</b>	
Muscle longitudinal ( $\sigma_l$ )	0.40 S m <sup>-1</sup>
Muscle transversal ( $\sigma_t$ )	0.09 S m <sup>-1</sup>
Fat ( $\sigma_l = \sigma_t$ )	4.07 · 10 <sup>-2</sup> S m <sup>-1</sup>
Skin ( $\sigma_l = \sigma_t$ )	4.88 · 10 <sup>-4</sup> S m <sup>-1</sup>
<b>Action potential parameters calculated with [7]</b>	
Propagation Velocity $v$	2.5 m s <sup>-1</sup>
Amplitude ( $\sigma_l \cdot \pi \cdot r^2$ )	3.72 · 10 <sup>-10</sup> S m

MATLAB these factors along the fiber result in the so-called weighting function. The calculation of the SFAP by means of the weighting function is subsequently performed as suggested by *Duchene and Hogrel* [10] with  $\Delta t = 1.478 \times 10^{-5}$  s.

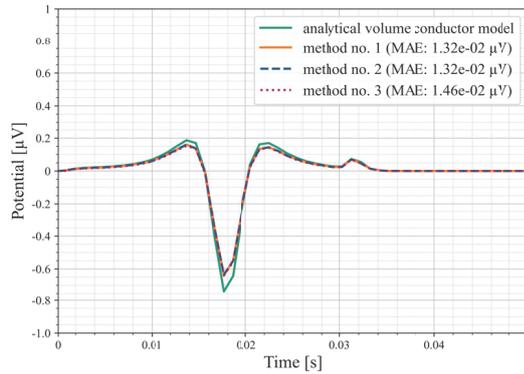
## 2.4 SFAP simulation

The analytical and numerical simulations of the SFAP are performed with 1024 Hz and the model properties listed in Table 2. To increase the accuracy of the calculation, the area around the fiber is meshed more finely. In general, the identical meshing is used for all numerical models. The simulation for the considered fiber is performed on Intel® Core™ i7-10850H CPU @ 2.70GHz 2.71GHz. The mean absolute error (MAE) is calculated for comparison of analytical and numerical results.

## 3 Results

The simulation results of all three numerical methods compared to the solution with the analytical volume conductor model are shown in Figure 2. The three numerical methods no. 1 to no. 3 have highly similar potential curves of the SFAP for the investigated electrode position. The propagating signal part as well as the end-of-fiber effect are clearly visible. The MAE to the solution with the analytical volume conductor model (0 s - 0.04 s) increases from  $1.32 \times 10^{-2}$   $\mu$ V for method no. 1 and  $1.32 \times 10^{-2}$   $\mu$ V for method no. 2 to  $1.46 \times 10^{-2}$   $\mu$ V for method no. 3. A deviation between SFAP calculation with the analytical model and the solution of the numerical methods of 0.1  $\mu$ V is visible in the negative peak of the propagating signal component. For the end-of-fiber effect, the deviations of the signals are considerably lower.

The effect of the methods on the calculation time is listed in Table 3. From method no. 1 to method no. 3, the calculation duration per fiber decreases substantially. For method no. 2 and no. 3, there is an additional time period required for the calculation per electrode which also decreases.



**Fig. 2:** Simulated SFAP for the three numerical methods each compared to the analytical solution (green). The mean absolute error (MAE) between 0 s and 0.04 s is calculate to compare the accuracy between the methods of the acceleration pipeline

**Tab. 3:** Calculation time for the three numerical methods per electrode and per fiber

Method	Calculation time per electrode [s]	Calculation time per fiber [s]
No. 1	-	823
No. 2	345	1.7
No. 3	14	0.43

## 4 Discussion

The three numerical methods for the simplified muscle geometry presented allow the SFAP to be determined. The difference to the solution with the analytical volume conductor model is acceptable in context of sEMG modelling. The deviation can be explained by the assumptions made in the transfer of the analytical volume conductor to the numerical model, as well as by the used approximations such as numerical integration in [4] and the finite element method in method no. 1 to method no. 3. The only slight increase in mean absolute error from method no. 1 to no. 3 indicates that the assumptions made in the acceleration pipeline are reasonable. This confirms the application of the reciprocity principle for the calculation of the SFAP proposed in [3]. The small difference between method no. 2 and method no. 3 also shows that the low-pass behavior can be neglected for the applied model properties.

The long calculation time of the numerical models is also evident in the case of model no. 1. In practice, a simulation duration of 823 s per fiber is not feasible for muscles with a high number of fibers. Numerical method no. 2 and no. 3 are proportional to the number of electrodes in addition to the number of fibers. Nevertheless, they allow a significant acceleration of the SFAP calculation while maintaining the same

meshing, which makes them feasible in practice, as proposed in [3]. The simplified muscle model presented here has been selected for verifying the acceleration pipeline. However, the real factor for speeding up the computation time depends on the accurate anatomical geometry with a high number of muscle fibers chosen in the future.

### Author Statement

**Research funding:** This work was supported by European Union – European Regional Development Fund, the Federal Government and Land Schleswig-Holstein, Project: “Diagnose- und Therapieverfahren für die Individualisierte Medizintechnik (IMTE)“, Project No. 12420002. **Conflict of interest:** Authors state no conflict of interest. **Ethical approval:** The conducted research is not related to either human or animal use.

## References

- [1] Merletti R, Farina D. Surface Electromyography: Physiology, Engineering, and Applications. Hoboken: John Wiley & Sons, Inc; 2016.
- [2] Graßhoff J, Petersen E, Farquharson F, Kustermann M, Kabitz H-J, Rostalski P, Walterspercher S. Surface EMG-based quantification of inspiratory effort: a quantitative comparison with Pes. *Critical Care* 2021;25:441.
- [3] Pereira Botelho D, Curran, K, Lowery, MM. Anatomically accurate model of EMG during index finger flexion and abduction derived from diffusion tensor imaging. *PLoS computational biology* 2019;15(8):e1007267.
- [4] Petersen E, Rostalski P. A Comprehensive Mathematical Model of Motor Unit Pool Organization, Surface Electromyography, and Force Generation. *Frontiers in Physiology* 2019;10:176.
- [5] Lowery MM, Stoykov NS, Kuiken TA. A simulation study to examine the use of cross-correlation as an estimate of surface EMG cross talk. *Journal of Applied Physiology* 2003;94(4):1324-1334.
- [6] Farina D, Merletti R. A Novel Approach for Precise Simulation of the EMG Signal Detected by Surface Electrodes. *IEEE Transactions on Biomedical Engineering* 2001;48(6):637-646.
- [7] Petersen E. GitHub. semgsim: A simulation model of surface electromyography (sEMG) and force generation. <https://github.com/ime-luebeck/semgsim>. 2021.
- [8] Plonsey R, Barr R C. Bioelectricity: A Quantitative Approach. 3rd ed. Bosten: Springer; 2007.
- [9] Rosenfalck R. Intra- and extracellular potential fields of active nerve and muscle fibres. A physico-mathematical analysis of different models. *Acta Physiologica Scandinavica* 1969;Suppl 321:1–168.
- [10] Duchene J, Hogrel JY. A Model of EMG Generation. *IEEE Transactions on Biomedical Engineering* 2000;47(2):192-201.